Comparison of Digital Control Loops
Analytical Models, Laboratory Measurements, and Simulation Results

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Never stop thinking
Outline

- Application Circuit & IC Block Diagram
- Control Loop Model, Design, and Analysis
- PID Design – Analytical Design Procedure
- Simulation & Experimental Circuit Schematics
- Time-Domain Simulation Model vs. Experimental Results
- Frequency Domain Comparison
- Summary
Control Loop Model: Mostly Small-Signal

- Line-to-Output Transfer Function
- Output Impedance Transfer Function
- Lumped Total Delay
- Can include delay from DPWM block
- Feedback Gain: 1, 1/2, 1/3
- Control-to-Output Power Converter Averaged Model

Transfer functions in Continuous “s” or Discrete “z” frequency domains

Line-to-Output Transfer Function:
- $I_o(s)$
- $V_{in}(s)$

Controller:
- $G_c(s)$
- $G_c(z)$

DPWM:
- $G_{DPWM}(s)$
- $G_{DPWM}(z)$

Converter:
- $G_{OC}(s)$
- $G_{OC}(z)$

Output Impedance Transfer Function:
- $Z_o(s)$

Feedback Gain:
- $G_{OIN}(s)$

Lumped Total Delay:
- $e^{-s \cdot t_d}$

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Control Loop Design – What Do We Want To Do

Loop Gain

Control-to-Output

Controller

Crossover Frequency

Phase Boost

-180°
Control Loop Design – What Do We Want To Do

Loop Gain

Control-to-Output

Controller

Crossover Frequency

Phase Boost and Gain Adjust

-180°
Analysis: Small-Signal Equations

Total Discrete Plant and Feedback
These values \((n_1, n_0, d_1, d_0, H)\) are known

Discrete Controller
\[
G_C(z) = A \frac{az^2 + bz + c}{z^2 - (1 + K_{FD})z + K_{FD}} = A \left[ K_I \cdot \frac{1}{1 - z^{-1}} + \left( K_P + K_D \cdot |1 - z^{-1}| \right) \frac{1}{1 - K_{FD} z^{-1}} \right]
\]

Loop Gain
\[
|T(z)| = |G_C(z) G_{ZI}(z)|
\]

Solution:
Solve at Crossover
\[
20 \log \left( |G_C(z_C) G_{ZI}(z_C)| \right) = 0 \quad \text{or} \quad |G_C(z_C) G_{ZI}(z_C)| = 1
\]
PID Design – Analytical Design Procedure

1. Select desired analog crossover frequency $f_C$, this is the loop bandwidth, and calculate system resonance $f_o$ from the power converter reactive components

2. Set the “analog” post filter pole, $f_{PA2}$, to $3 \cdot f_C$, and find $K_{FD}$
   - A reasonable starting range is from $f_{PA2} = f_C / 2$ to $3 \cdot f_C$
   - $K_{FD}$ is one of the following {0.125, 0.25, 0.375, 0.50, 0.625, 0.75, 0.875, 1.00} for the PX7510D

3. Start with $f_X = 0.85 \cdot f_o$ and $Q_X = 0.7$ for the controller zeroes and find the required loop-gain (i.e., find $\alpha$) to have $T(z)$ crossover at $f_C$
   - $f_X$ should be equal to or less (for design margin) than $f_o$, but not too low

4. Find $\alpha$ from:

$$G_{Z1}(z) = \frac{n_1 z + n_0}{z^2 + d_1 z + d_0} H$$

Using

$$\alpha = \frac{|n_1 z_C + n_0|}{|z_C^2 + d_1 z_C + d_0|} H$$
PID Design – Analytical Design Procedure

- Where $G_{Z_1}$ is the total discrete plant and feedback gain

5. From the discrete controller transfer function, find $\beta$

$$G_C(z) = A \frac{az^2 + bz + c}{z^2 - (1 + K_{FD})z + K_{FD}}$$

Find $\beta$

$$\beta = \left| \frac{z_C^2 - (1 + K_{FD})z_C + K_{FD}}{z_C} \right|$$

6. Using pole-zero mapping ($z = e^{sT}$), along with the discrete crossover $z_C$, find $\gamma$

$$z = e^{sT}$$

$$z_C = e^{-j\omega_C T_S}$$

$$w_C = 2\pi f_C$$

analog maps to digital

$$1 + \frac{s}{Q_X}w_X + \left(\frac{s}{w_X}\right)^2 \rightarrow (z - z_{ZN1})(z - z_{ZN2})$$

Find $\gamma$

$$\gamma = \left| z_C - z_{ZN1} \right| \left| z_C - z_{ZN2} \right|$$

7. Solve for $r$ using $f_X$ and $Q_X$ in:

$$r = e^{-\pi f_X T_S / Q_X}$$
8. Finally the $a$, $b$, and $c$ controller terms are:

$$a = \frac{\beta}{\alpha \cdot \gamma \cdot A}$$

$$b = -2 \cdot a \cdot r \cdot \cos \left[ 2 \cdot \pi \cdot f \cdot x \cdot T \cdot s \cdot \sqrt{1 - \frac{1}{2 \cdot Q \cdot x^2}} \right]$$

$$c = a \cdot r^2$$

9. Alternatively, the $K_P$, $K_I$, and $K_D$ terms are:

$$K_D = c$$

$$K_I = \frac{a + b + K_D}{1 - K_{FD}}$$

$$K_P = a - K_I - K_D$$
Experiment Circuit Schematic

Integrated Driver and MOSFETs (PX4660)

Latest PX7510D Controller
Time-Domain Simulation vs. Experimental Results

5 A to 10 A Load Step

SIMPLIS Simulation Model

Imported Scope Data
Time-Domain Simulation vs. Experimental Results

10 A to 5 A Load Step

SIMPLIS Simulation Model

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Time-Domain Simulation vs. Experimental Results

5 A to 20 A Load Step

SIMPLIS Simulation Model

Imported Scope Data

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Time-Domain Simulation vs. Experimental Results

20 A to 5 A Load Step

SIMPLIS Simulation Model

Imported Scope Data
Experiment Results: Time-Domain

Actual Scope Plots
All data was extracted to .csv file for comparison

5 A to 10 A Load Step

10 A to 5 A Load Step
Frequency-Domain Comparison: Original Design

fsw is the Switching Frequency

The MatLab model shown here uses a more accurate digital loop model.
Frequency-Domain Comparison: Original Design

The MatLab model shown here uses a simplified digital loop model.

Both the gain and phase are less accurate at the higher frequencies.

- SIMPLIS T
- MatLab T
- Measured T
This is a more aggressive design.

The MatLab model shown here uses a more accurate digital loop model.

More phase boost throughout, higher crossover achievable.
Summary

Understanding Digital Control systems requires control loop models - The behavior can be better appreciated by analytical analysis aided with computer simulation tools in the time and frequency domain to gain further insight.
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- This represents a digital design example where all of the results are compared – this provides confidence that these systems are understood and designs can be robust using these approaches.